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HOT ELECTRON DISTRIBUTION FUNCTION IN HIGH-Z MATERIALS

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ABSTRACT

An analytic expression is obtained for the distribution of hot electrons as a function of distance into a cold material slab in which $\nu_E = \gamma_E v^{-\alpha}$ and $\nu_\perp = \gamma v^{-\beta}$ are the assumed velocity dependences of the energy loss collision frequency and the scattering collision frequency and $\nu_E \ll \nu_\perp$. For a time independent source, a time independent solution exists with a characteristic penetration length equal to $(\lambda_E \lambda_\perp)^{1/2}$ where λ_E and $\lambda_\perp$ are the mean-free-paths corresponding to ν_E and $\nu_\perp$. The distribution function at large distance from the source is proportional to $v^{\alpha-3}$ for v less than a critical velocity. For v greater than the critical velocity, the velocity dependence of the Maxwellian source reappears causing an exponential cutoff.

I. Introduction

The general problem of energy transport in a laser-produced plasma is sufficiently complex to require the use of sophisticated numerical techniques[1-4] to calculate it. However, certain key features may often be exhibited in analytic calculations of a restricted class of problems. Here we treat the problem of hot electron transport from a plane source into a uniform isotropic medium in which the scattering and slowing down collision frequencies are assumed to be simple power-law functions of the individual electron energy. An analytic expression for the velocity distribution as a function of position is obtained for an arbitrary volume source of hot electrons. The special case of a source with a Maxwellian velocity distribution and an x-dependence given by a Dirac delta function is pursued in more detail.

II. Derivation of Equations

We start with the Fokker-Planck equation

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \nabla f + \underline{a} \cdot \nabla_v f = \left(\frac{\partial f}{\partial t} \right)_{coll} + S \qquad (1)$$

where $(\partial f/\partial t)_{coll} = -\sum_j \nabla_v \cdot \underline{J}_j$ is the collision integral and $\underline{J}_j$ may be written

$$\underline{J}_j = 4\pi\lambda_j \frac{e^2 e_j^2}{m^2} \left\{ f(\underline{v})\nabla_v H(\underline{v}) - \frac{1}{2} \nabla_v \cdot [f(\underline{v})\nabla_v \nabla_v G(\underline{v})] \right\} \qquad (2)$$

where

$$G(\underline{v}) = \int f_j(\underline{v}') \, |\underline{v}' - \underline{v}| \, d^3v'$$

$$H(\underline{v}) = \left(1 + \frac{m}{m_j} \right) \int f_j(\underline{v}') \, |\underline{v}' - \underline{v}|^{-1} \, d^3v'$$

are the Rosenbluth potentials.[5] If the hot electron density is small compared to the density of the plasma through which they are streaming, their interaction with each other may be ignored and $\underline{J}_j$ reduces to

$$\underline{J}_j = -\frac{1}{2}\nu_{Ej}\underline{v}f \quad \nu_{\perp j} v^2 \nabla_v f - \frac{1}{2}(\nu_{\perp j} - \nu_{\parallel j})\,\underline{vv} \cdot \nabla_v f \tag{3}$$

where ν_{Ej}, $\nu_{\perp j}$, and $\nu_{\parallel j}$ are the energy loss collision rate, the perpendicular velocity diffusion rate and the parallel velocity diffusion rate for hot electrons interacting with species j. Assume $f = f(x,v,\mu,t)$ where x is the spatial coordinate, v is the magnitude of the velocity $\mu = \cos\theta$, and θ is the angle between the x-axis and the velocity direction. The Fokker-Planck equaiton then becomes

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial x} + a\mu \frac{\partial f}{\partial v} + \frac{1-\mu^2}{v}\frac{\partial f}{\partial \mu} =$$

$$+ \sum_j \left\{ \frac{1}{2v^2}\frac{\partial}{\partial v}(v^3 \nu_{Ej} f) + \frac{1}{2v^2}\frac{\partial}{\partial v}\left(v^4 \nu_{\parallel j}\frac{\partial f}{\partial v}\right) \right. \tag{4}$$

$$\left. + \frac{1}{2}\nu_{\perp j}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial f}{\partial \mu}\right]\right\} + S \quad .$$

And, finally, we assume that $\nu_\perp \gg \nu_E \gg \nu_\parallel$, and expand f in a series of Legendre functions, $f = \sum_\ell f_\ell(x,v,t)P_\ell(\mu)$. The equations for $\ell = 0$ and $\ell = 1$ are

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\frac{\partial f_1}{\partial x} + \frac{a}{3}\frac{\partial f_1}{\partial v} + \frac{2a}{3v} f_1 = \frac{1}{2v^2}\frac{\partial}{\partial v}(v^3 \nu_E f_0) + S \tag{5}$$

$$v\frac{\partial f_0}{\partial x} + a\frac{\partial f_0}{\partial v} = -\nu_\perp f_1 \tag{6}$$

where the time derivative in the second equation has been assumed small compared to $\nu_\perp$.

The ordering of the collision frequencies assumed above is appropriate for intermediate and high z materials when the effects of the bound electrons are included. Expressions for these collision frequencies differ from the more familiar forms for interacting point charges in the factors ($\lambda_j = \ell n \Lambda_j$) which account for the screening of the Coulomb potential. Formulae for the scattering rate, for example, may be obtained from Ref. 6. For the energy loss rate, Gene McCall[7] has obtained convenient and accurate expressions by fitting the data in Ref. 8 with a power law. Taking Au as an example, we have

$$\nu_\perp = \left(\frac{\rho}{19.3}\right)\left(\frac{2.9\times 10^{44}}{v^3}\right) \ell n(8.6\times 10^{-10} v)$$

$$\nu_E = \left(\frac{\rho}{19.3}\right)\left(\frac{3.0\times 10^{35}}{v^{2.2}}\right)$$

with collision frequencies in sec^{-1} and velocities in cm/sec. At 10 keV, for instance $\nu_E/\nu_\perp \approx 0.04$ which is consistent with the ordering assumed above.

At this point we neglect the velocity dependence in the log term and use the following expressions for the collision frequencies:

$$\nu_E = \gamma_E v^{-\alpha}$$

$$\nu_\perp = \gamma_\perp v^{-\beta} \quad .$$

Using Eq. (6) to eliminate f_1 in Eq. (5) we obtain:

$$\frac{\partial}{\partial v}\left(v^{3-\alpha} f\right) + \frac{2v^{\beta+4}}{3\gamma_\perp \gamma_E}\frac{\partial^2 f}{\partial x^2} - \frac{2v^2}{\gamma_E}\frac{\partial f}{\partial t} = -S' \qquad (7)$$

where

$$S'(x,v,t) = \frac{2v^2}{\gamma_E} S(x,v,t) \tag{8}$$

Using a Laplace transform in time and a Fourier transform in space, Eq. (7) may be solved for f.

$$f = \frac{v^{\alpha-3}}{\pi^{1/2}} \int_{-\infty}^{\infty} \int_{v}^{w} K \exp[-K^2(x-y)^2]\, S'\left(y,u,t - 2\,\frac{u^\alpha - v^\alpha}{\alpha\gamma_F}\right) du\,dy \tag{9}$$

where

$$w = \left(v^\alpha + \frac{1}{2}\alpha\gamma_F t\right)^{1/\alpha}$$

and

$$K = \left[\frac{\delta\gamma_F\gamma_\perp}{8(u^\delta - v^\delta)}\right]^{1/2}$$

$$\delta = \alpha + \beta + 2$$

The expression for f obtained in this manner is exact for zero electric field ($a = eF_x/m = 0$), and the effects of a weak electric field may be included by iteration.

Because laser pulse lengths and hydro time scales tend to be long compared to collision times at near solid densities, the time dependence in the source may be ignored and the limit, $t \to \infty$, may be taken. Assuming a source whose space dependence is $\delta(x)$, we obtain

$$f = v^{\alpha-3}\pi^{-1/2} \int_v^{\infty} K \exp(-K^2x^2)\, S'(u)\, du \qquad (10)$$

where

$$S'(u) = 2\nu\gamma_E^{-1}u^2 g(u) \quad .$$

ν is a hot electron creation rate and $g(u) = (2\pi)^{-3/2} \; v_H^{-3} \; \exp[-u^2/(2v_H^2)]$ for the special case of a Maxwellian source.

III. Numerical Results

A fortran program, shown in the Appendix, has been written to evaluate the distribution function using Eq. (10). As a specific example we consider gold and a hot electron temperature of 10 keV. We choose $v_H = 1$, and $(\nu_E\nu_\perp)^{1/2} = 1$ to define the dimensionless units used in the calculation. The ratio of the collision frequencies is 50 at 10 keV, therefore we pick γ_E + GE = 0.2 and $\gamma_\perp$ + GP = 5.0. The dimensionless unit of length defined in this manner corresponds to 0.041 μm at solid density (19.3 g/cm^3).

The velocity distribution of the Maxwellian source is shown in Fig. 1. Its spatial dependence is, of course, a Dirac delta function at x = 0. Figures 2-4, show the computed value of f as a function of x for v = 0.24, 1.04, and 3.04. As expected the more energetic particles have a broader spatial distribution because of their greater range and smaller energy loss rate.

The distribution function, f, as a function of v for x = 0, 1.1, and 5.5 is shown in Figs. 5 to 7. The Maxwellian velocity dependence of the source is no longer present. The distribution, in fact, is dominated by the $v^{\alpha-3}$ factor in front of the integral. To show this more clearly $v^{3-\alpha}f$ is plotted vs v in Figs. 8 to 11. A function which is flat from v = 0 out to some critical v is obtained. For larger v the velocity dependence of the source generates an exponential cutoff

IV. Discussion and Conclusions

The interaction of hot electrons with a background plasma is essentially a linear problem which can be solved using transform methods. The resulting velocity distribution is far from a Maxwellian. The dominant velocity dependence is a power law, e.g. $v^{\alpha-3}$, with the power closely related to the velocity

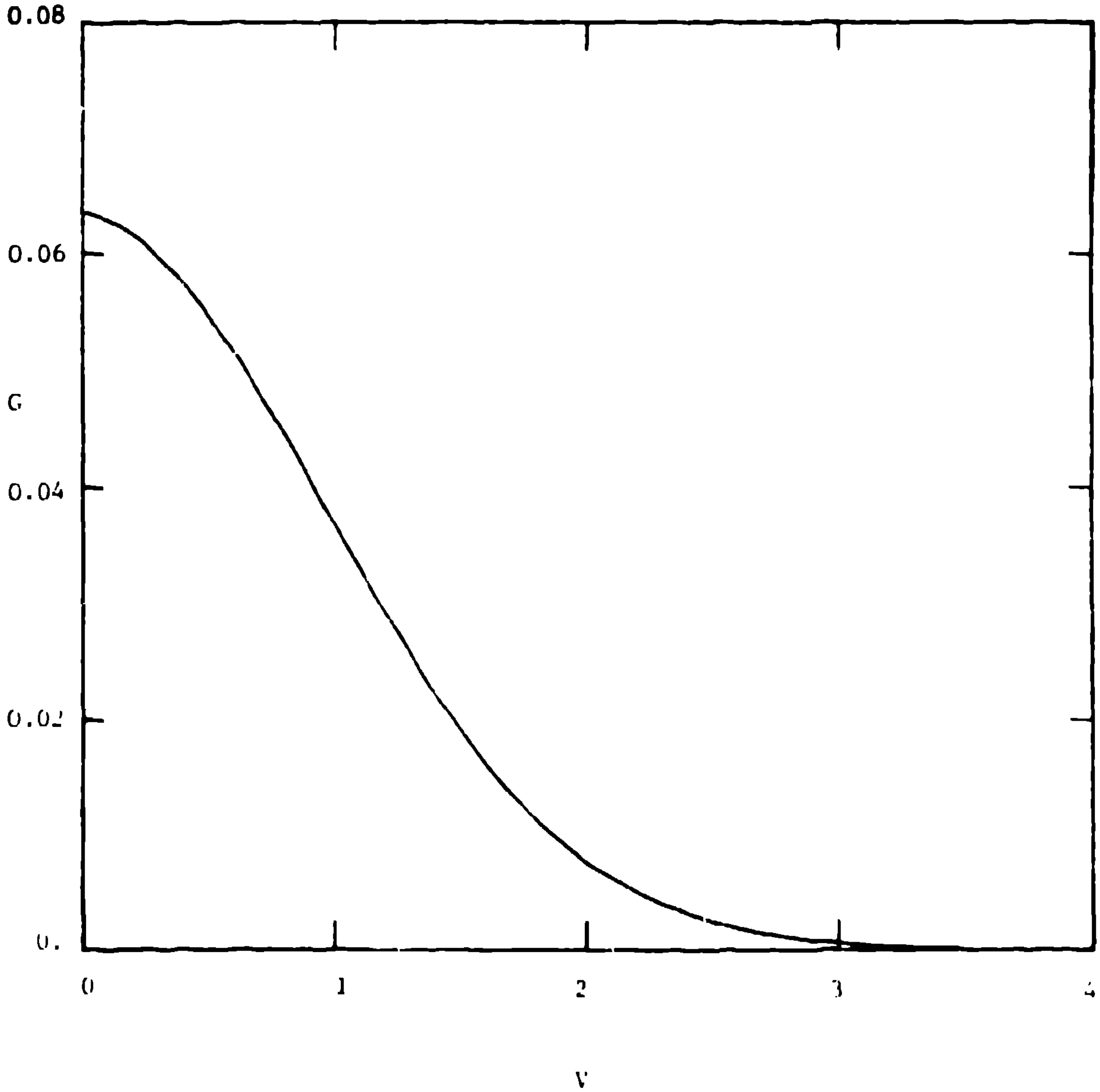

Fig. 1. The velocity distribution of the source is plotted as a function of v. Its spatial dependence is a Dirac delta function at x = 0.

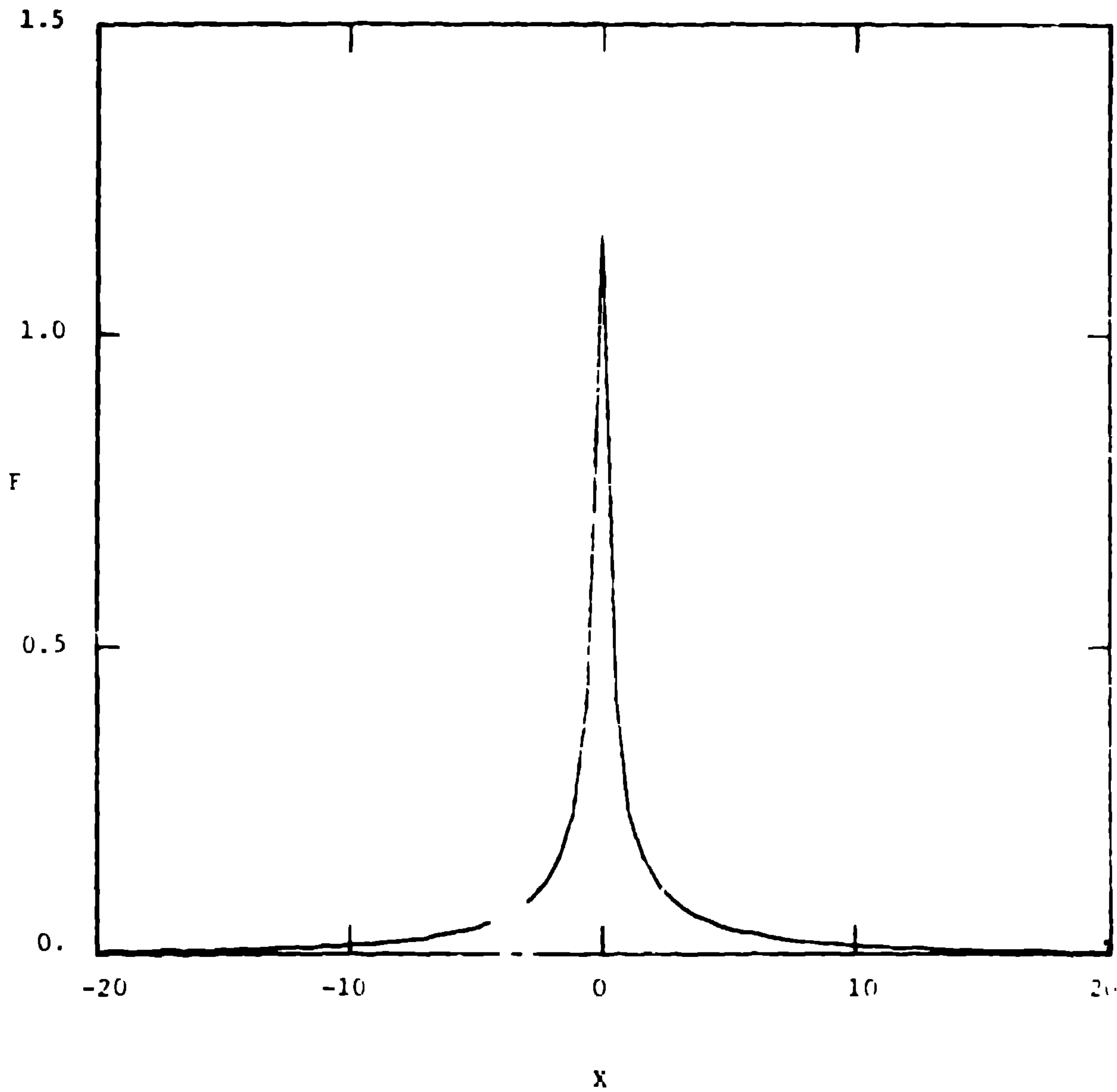

Fig. 2. The computed distribution function at v = 0.24 is plotted as a function of position.

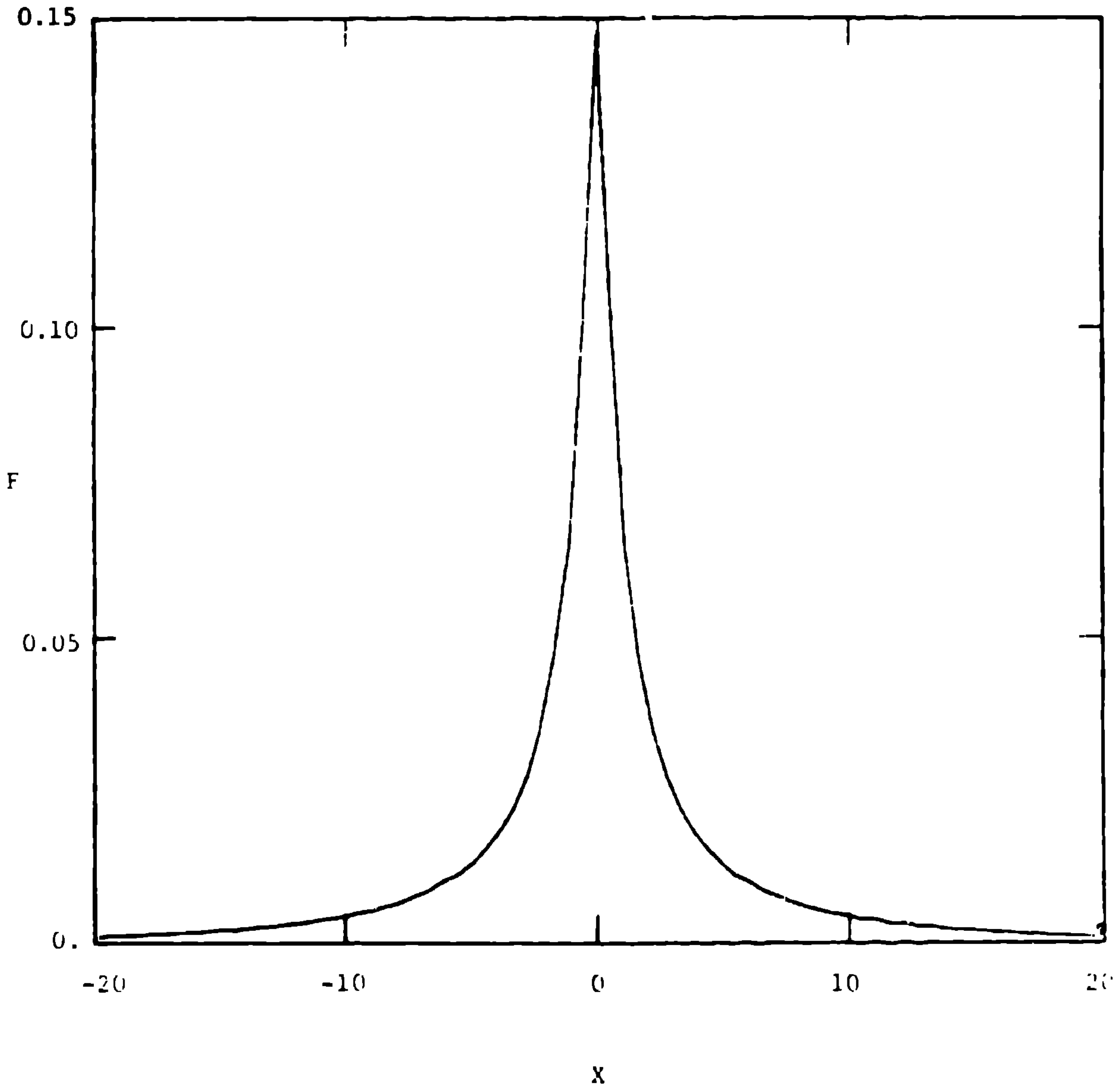

Fig. 3. The computed distribution function at $v = 1.04$ is plotted as a function of position.

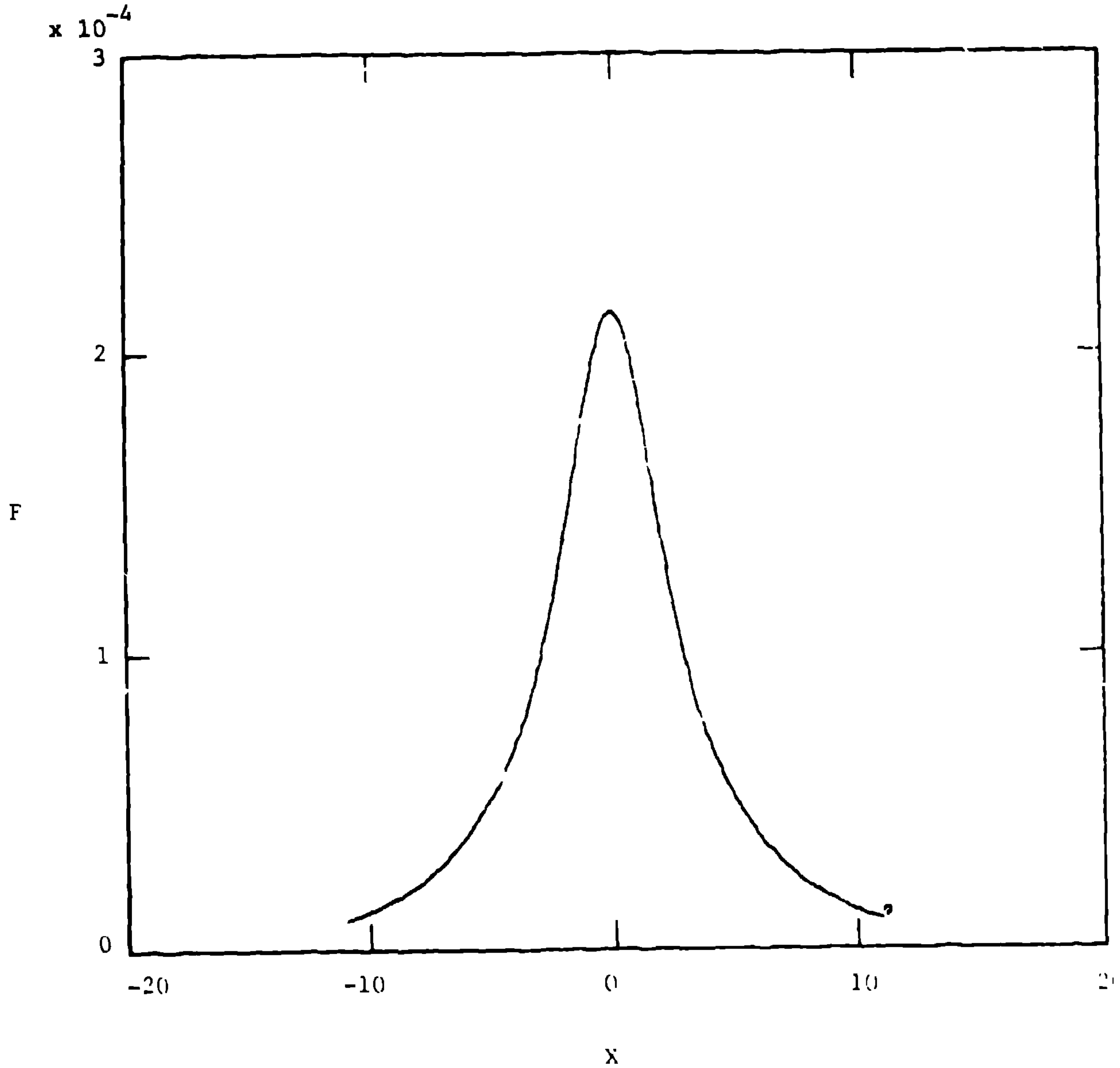

Fig. 4. The computed distribution function at $v = 3.04$ is plotted as a function of position.

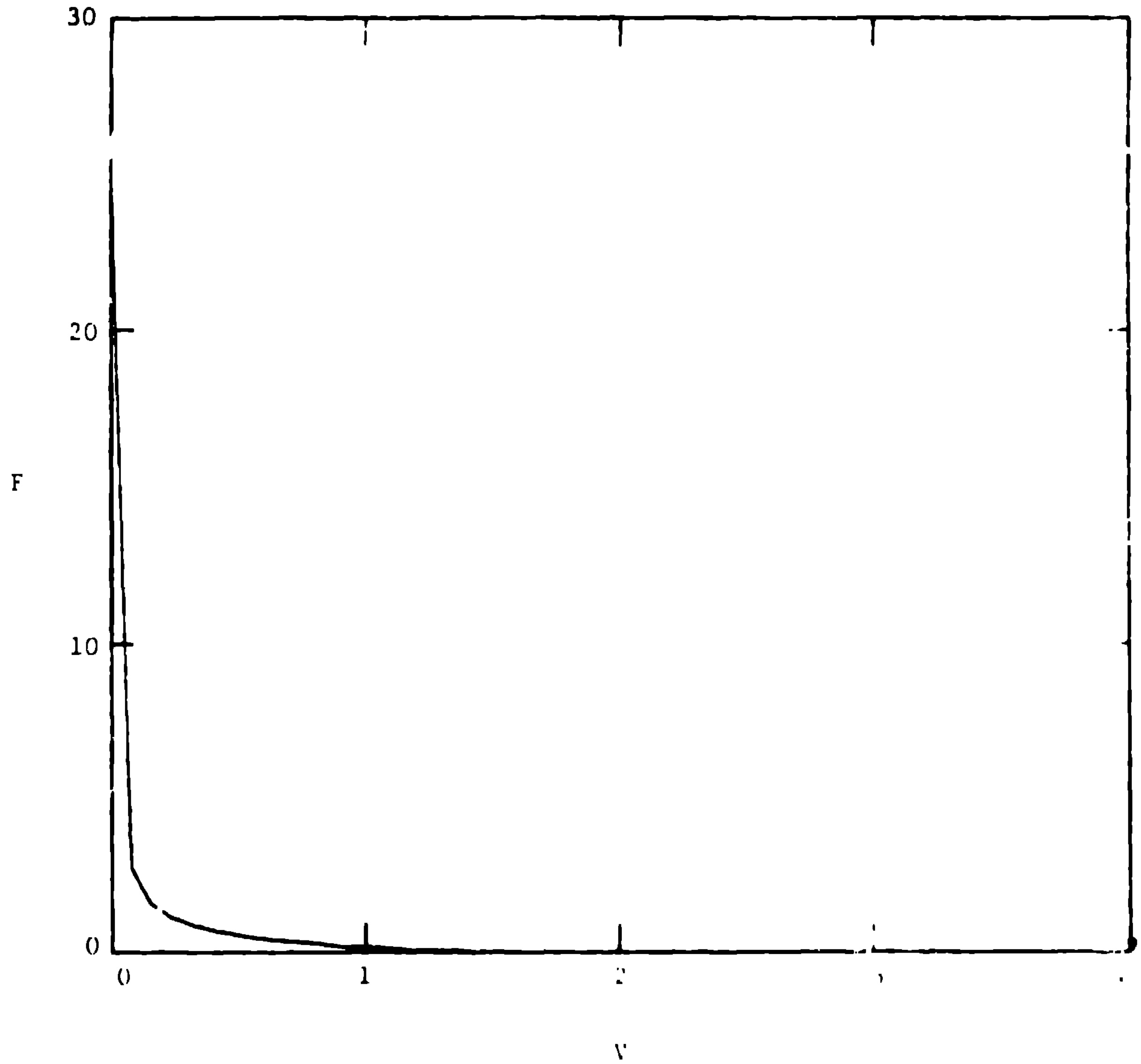

Fig. 5. The computed distribution function at $x = 0$ is plotted
as a function of velocity.

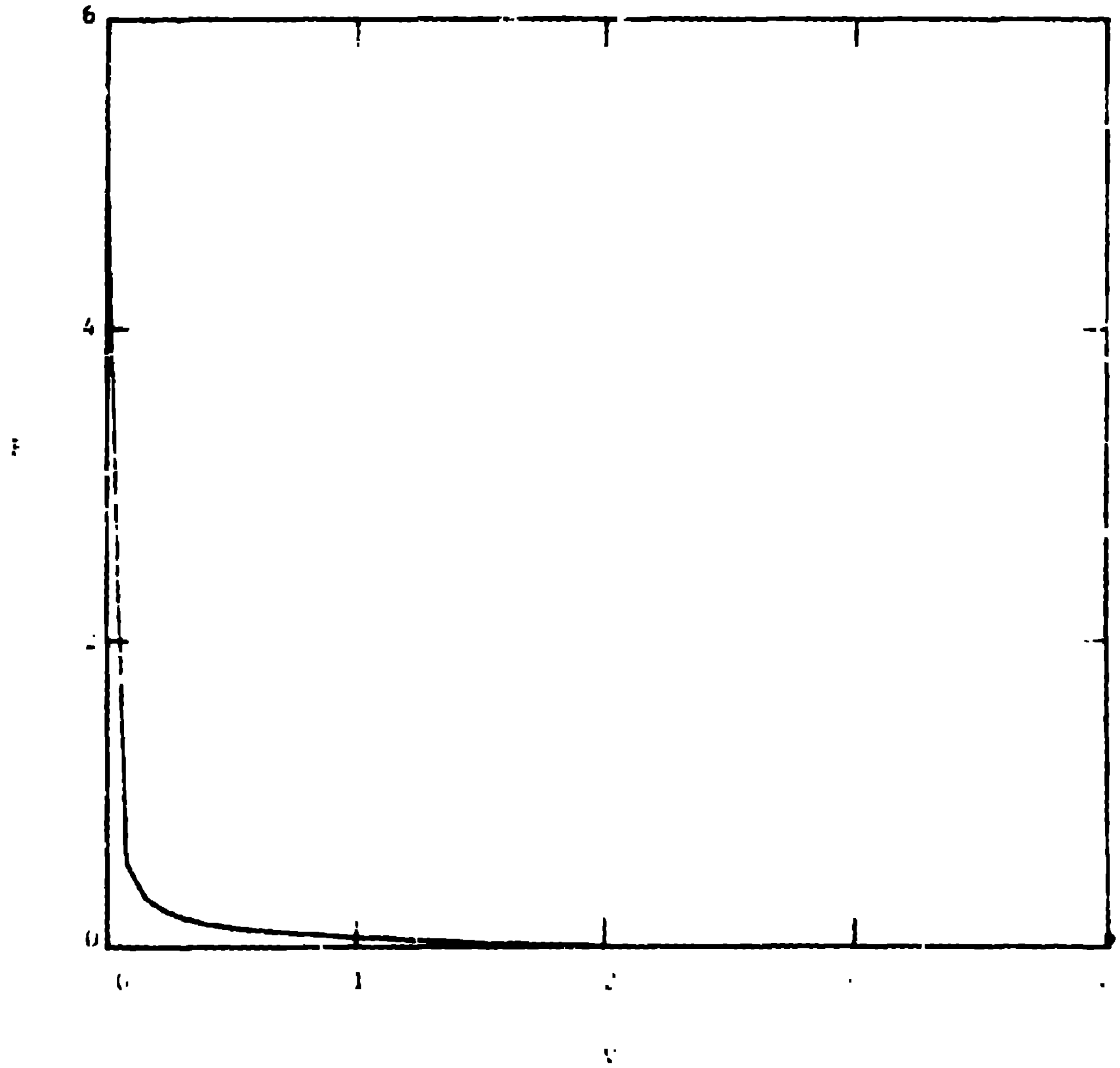

Fig. 6. The computed distribution function at x = 1.1 is plotted as a function of velocity.

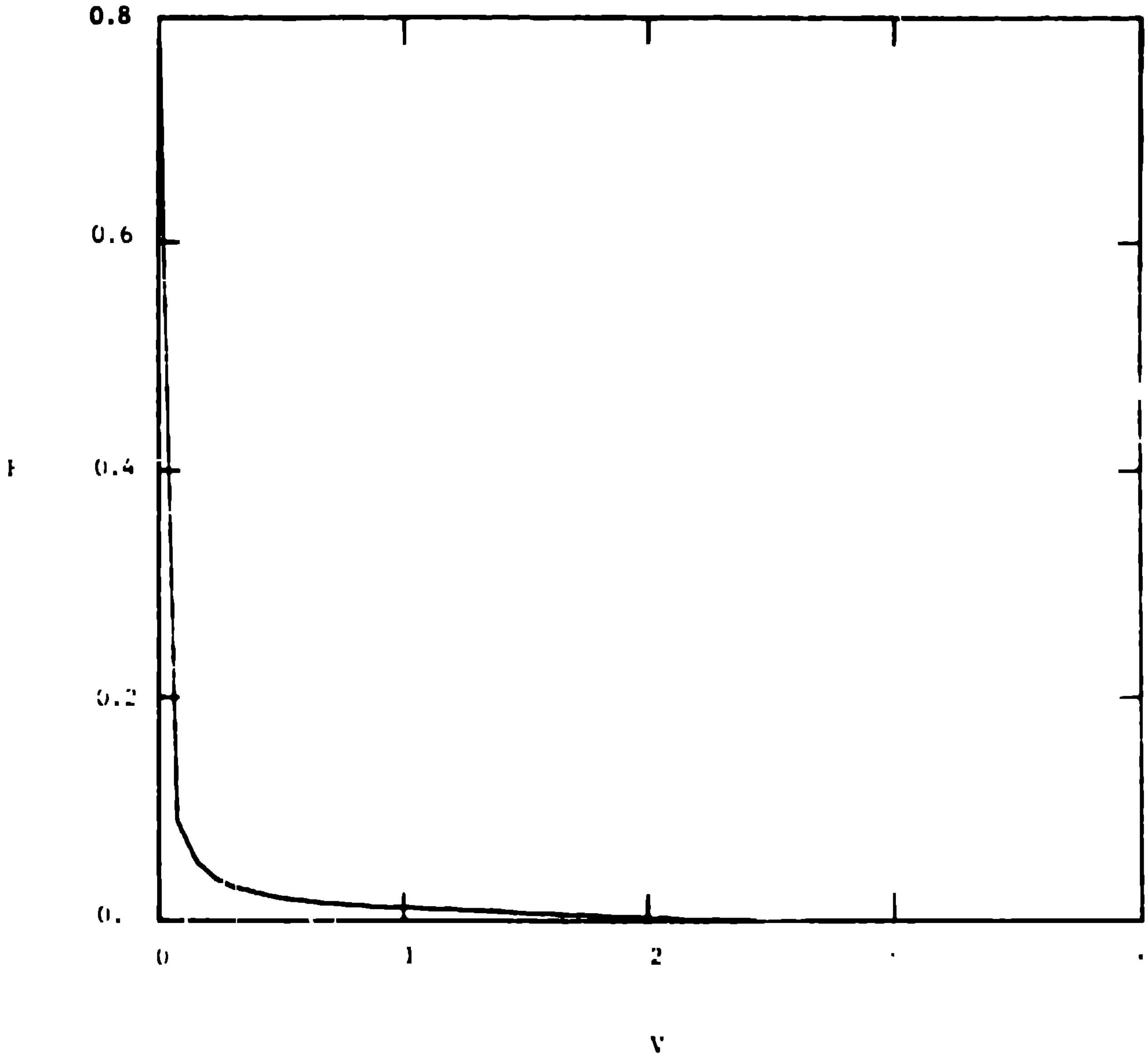

Fig. 7. The computed distribution function at x = 5.5 is plotted as a function of velocity.

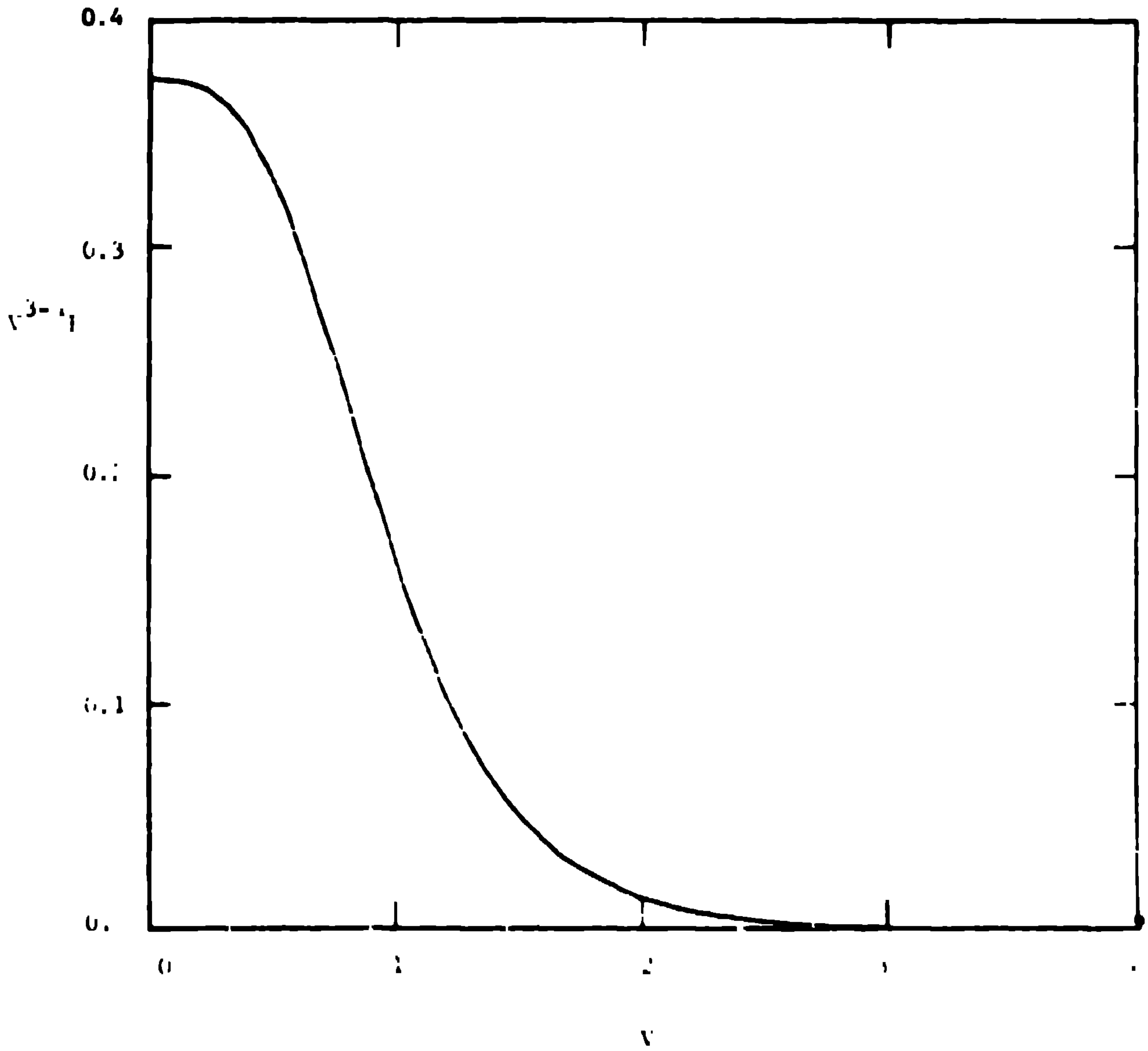

Fig. 8. The renormalized distribution function, $v^{3-\mu}f$, at x = 0 is plotted as a function of velocity.

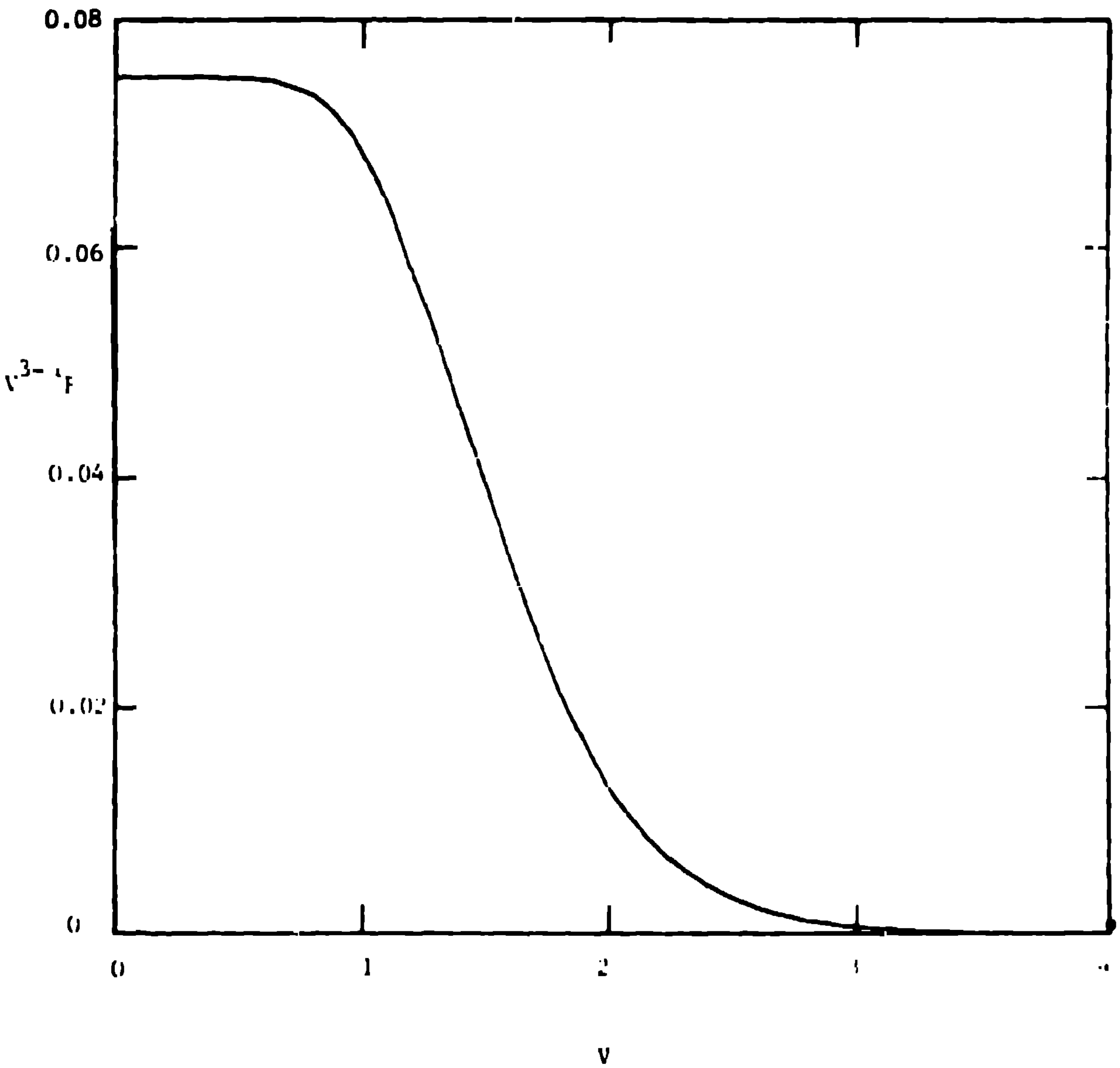

Fig. 9. The renormalized distribution function, $v^{3-\mu}f$, at $x = 1.1$ is plotted as a function of velocity.

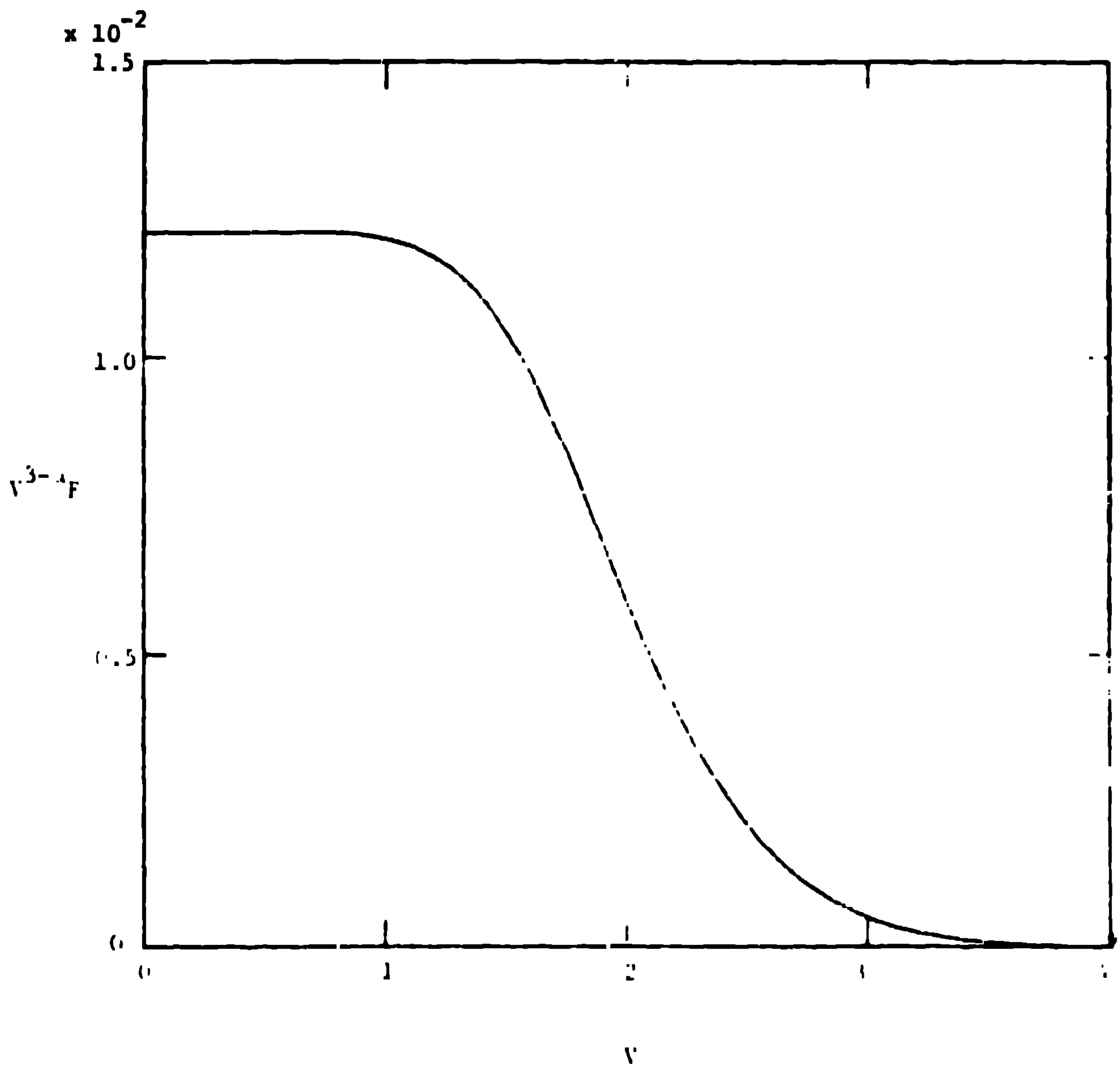

Fig. 10. The renormalized distribution function, $v^{3-\mu}f$, at $x = 5.5$ is plotted as a function of velocity.

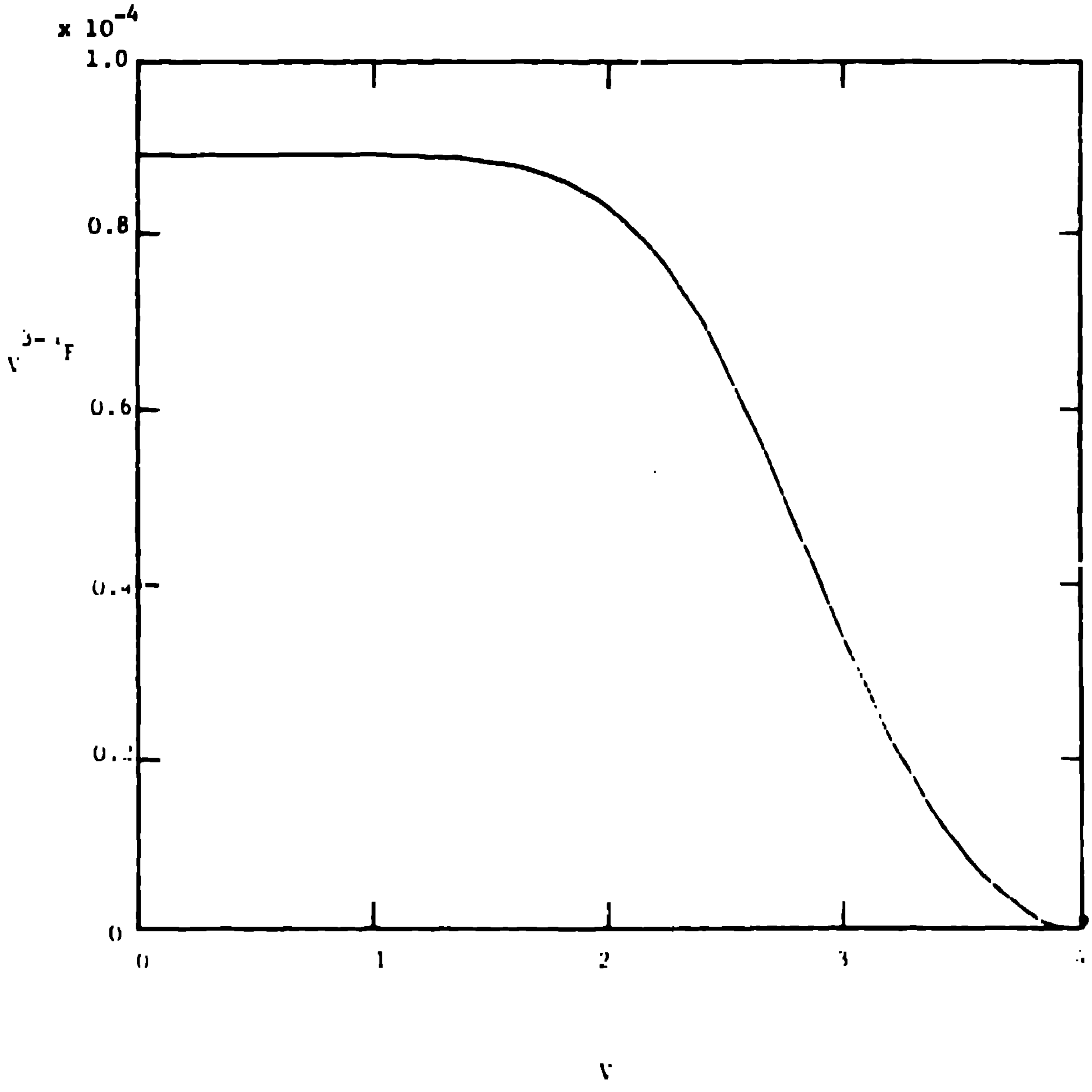

Fig. 11. The renormalized distribution function, $v^{3-\alpha}f$, at $x = 50$. is plotted as a function of velocity.

dependence of the energy loss rate. The scale length for the spatial fall off is the geometric mean of the energy loss mean-free-path and the scattering mean-free-path. A code has been written which can be used to evaluate a wide range of cases including an arbitrary velocity distribution for the source.

Extensions of this work which might be considered are the inclusion of electric field effects, and the elimination of the assumption that the hot electron energy is large compared to the temperature of the background plasma so that thermalization can be observed.

The inclusion of electric field effects makes the problem non-linear. Iterating with the linear operator defined in the previous section is a possible method for solving such problems. Such a procedure might converge slowly or not at all. Thus whether it would be numerically efficient compared to other methods[1-4] currently in use is not clear. In any case the results would probably be similar.

The second alternative is probably more interesting. The energetic electrons whether they are true hot electrons with a temperature moderately above the background or simply the tail of a Maxwellian as suggested by Shvarts, et al.,[2] will interact quite weakly with each other. The strong interaction is with a background plasma which will thermalize itself. Thus the problem remains linear and the added effects can be included by including the temperature of the background plasma in the collision frequencies and by retaining some of the terms neglected here.

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<u>References</u>

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2. Shvarts, D., Jablon, C., Bernstein, I. B., Virmont, J., and Mora, P., Nuclear Fusion <u>19</u>, 1457 (1979).
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8. McCall, G. H., Private Communication.

APPENDIX

```
      PROGRAM HTRAN(TTY)
      DIMENSION ITYPE(40),WORD(40),LINE(10)
      PARAMETER (NX=100,NV=50)
      PARAMETER (NX2P1=NX*2+1,NVP1=NV+1)
      DIMENSION F(NX2P1,NVP1),S(NX2P1,NVP1),XX(2),VV(2)
      DIMENSION FO(NVP1)
      DIMENSION LTITLE(2)
C
C  INITIAL VALUES
C
      VTH=1.
      GE=.2
      GP=5.
      ES=2.2
      EP=3.
      E8=ES+EP+2.
      HE8=.5*E8
      EO=ES-3.
      ENORM=-EO
      XMAX=(3.*VTH)**HE8/SQRT(.125*E8*GE*GP)
      VMAX=4.*VTH
      SNU=1.
      PI=2.*ASIN(1.)
      SQPI=SQRT(PI)
      SQ2PI=SQRT(2.*PI)
      NXP1=NX+1
      NX2=NX*2
      DX=XMAX/NX
      DV=VMAX/NV
      DO 4 IV=1,NVP1
      V=(IV-.5)*DV
    4 FO(IV)=EXP(-.5*(V/VTH)**2)/(SQ2PI*VTH)**3
      DO 8 I=1,NVP1
      DO 8 J=1,NX2P1
    8 F(J,I)=0.
C
C  EVALUATE S(V)
C
      DO 9 I=1,NVP1
      DO 9 J=1,NX2P1
    9 S(J,I)=0.
      DO 10 IV=1,NV
      V=(IV-.5)*DV
   10 S(NXP1,IV)=FO(IV)*V**2*SNU*2./(GE*DX)
C
C  EVALUATE F(V)
C
      DO 110 I=1,NVP1
      DO 110 J=1,NX2P1
  110 F(J,I)=0.
      DO 150 IV=1,NV
```

```
      V=(IV-1)*DV
      VEO=(.5*DV)**EO/(EO+1.)
      IF(IV.GE.2) VEO=V**EO
      DO 150 IU=IV,NV
      U=(IU-.5)*DV
      FK=SQRT(.375*E8*GE*GP/(U**E8-V**E8))
      FK=VEO*DV*ERF(FK*.5*DX)
      DO 120 IX=1,NX2P1
  120 F(IX,IV)=F(IX,IV)+FK*S(IX,IU)
      DO 150 IY=1,NX2
      FK=SQRT(.375*E8*GE*GP/(U**E8-V**E8))
      FK=VEO*DV*.5*(ERF(FK*(IY+.5)*DX)-ERF(FK*(IY-.5)*DX))
      NXY=NX2-IY
      DO 130 IX=1,NXY
  130 F(IX,IV)=F(IX,IV)+FK*S(IX+IY,IU)
      NXY=IY+1
      DO 140 IX=NXY,NX2P1
  140 F(IX,IV)=F(IX,IV)+FK*S(IX-IY,IU)
  150 CONTINUE
      DO 155 IV=1,NVP1
      DO 155 IX=1,NX2P1
  155 F(IX,IV)=AMAX1(0.,F(IX,IV))
C
C  ACCURACY CHECK
C
      SFDX=0.
      SFDXV=0.
      SSDXV=0.
      SVSDXV=0.
      DO 160 IX=1,NX2P1
  160 SFDX=SFDX+F(IX,1)
      SFDX=SFDX*DX*(EO+1.)*(.5*DV)**(-EO)
      DO 170 IV=1,NVP1
      V=(IV-1)*DV
      VEOI=(EO+1.)*(.5*DV)**(-EO)
      IF(IV.GE.2) VEOI=V**(-EO)
      DO 170 IX=1,NX2P1
      SFDXV=SFDXV+VEOI*F(IX,IV)
      SSDXV=SSDXV+S(IX,IV)
  170 SVSDXV=SVSDXV+(V+.5*DV)*S(IX,IV)
      SFDXV=SFDXV*DX*DV
      SSDXV=SSDXV*DX*DV
      SVSDXV=SVSDXV*DX*DV
      CH1=(SFDX-SSDXV)/SSDXV
      CH2=(SFDXV-SVSDXV)/SVSDXV
      WRITE('TTY',171) CH1,CH2
  171 FORMAT("INTEGRAL OPERATOR ERROR CHECKS",2E12.3)
      WRITE('TTY',172) SFDX,SSDXV,SFDXV,SVSDXV
  172 FORMAT("SUMS IN CHECK",2E12.3,5X,2E12.3)
C
C  PLOT RESULTS
C
```